



VISIONARY TUTORING

Year 12 Physics ATAR: Intro to Projectile Motion

Name: _____

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YEAR 12 ATAR Rules to follow:

- 1) Always give answers to 3 significant figures
- 2) Always give estimates to 2 significant figures
- 3) Always provide a direction for vector quantities

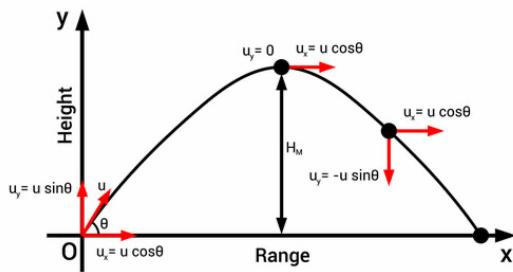
Analysing Projectile Motion:

ONLY 3 EQUATIONS WHICH HAVE TO BE USED IN PROJECTILE MOTION

$$1) v = u + at$$

$$2) v^2 = u^2 + 2as$$

$$3) s = ut + 0.5at^2$$



- Analysing the motion of projectiles is done by breaking the path of the projectile into its vertical and horizontal component

Vertical Motion:

- Here we ignore the effects of air-resistance therefore only acceleration of body in vertical axis is due to gravity with a magnitude of 9.8m/s^2 modelled by the equation:
- $v = u_0 \sin(\theta) - 9.8t$
- Where u_0 is the speed at which projectile is launched at and θ is the angle projectile is launched at to the horizontal
- Note: At "apex" vertical velocity is always 0m/s

Horizontal Motion

- As we ignore the effects of air-resistance, the acceleration of the body in the horizontal axis is simply 0 m/s^2 (since gravity only acts vertically downwards)
- $\therefore s = ut + 0.5at^2 \rightarrow s_x = u_0 \cos(\theta) + 0.5(0)t^2 \rightarrow s_x = u_0 \cos(\theta)$; where s_x is horizontal distance travelled



All Question Types:

Type 1: Horizontal Launch

Mr Taylor decided to one day test the limits of his green jeep and decided to drive his car off the edge of a cliff. Mr Taylor's car left the edge of the cliff perfectly horizontal with an initial velocity of 100m/s. If the height of the cliff was 150 metres

- a) Calculate Mr Taylor's flight time
- b) Calculate Mr Taylor's range
- c) Calculate Mr Taylor's final velocity before impact
- d) Calculate Mr Taylor's net displacement before impact



Type 2: Angle Launch

Mr Taylor woke up one morning feeling rather villainous... He decides he's going to cause some mayhem by shooting a missile towards Southlands from a bazooka at a velocity of 900km/h and at an angle of 45° to the horizontal. Southlands is 10km away from Mr Taylor's current location and assume no net vertical displacement travelled by missile

- a) Calculate the maximum height of the missile

- b) Calculate the total flight time of the missile

- c) Calculate the range of the missile; does the bazooka have the capability to hit Southlands

- d) Calculate the velocity of the missile when it's flight is $\frac{3}{4}$ complete

- e) Calculate final velocity of the missile

(Note: If performing any calculations you are on the wrong track)



Type 3: Angled Launch with net vertical displacement

This time Taylor decides to change the location he is shooting his bazooka and now stands on a cliff. If he launches a missile with an initial velocity of 150km/h and at an angle of 35 degrees to the cliff's surface. The cliff's height is 150m

- a) Calculate the maximum height above the sea reached by the missile

- b) Calculate the missile's total flight time

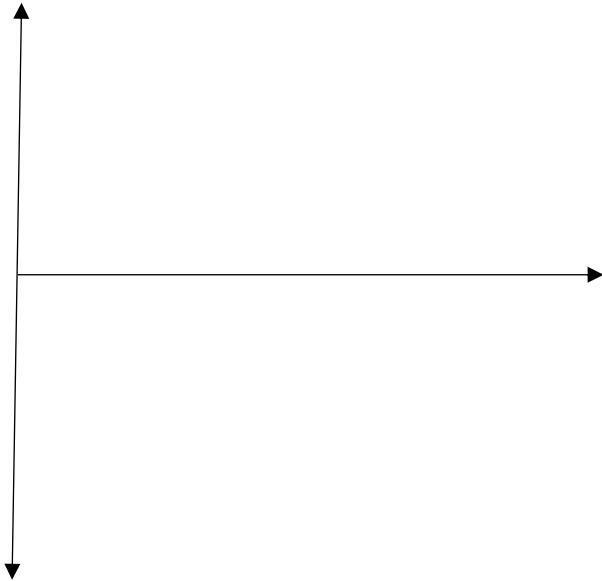
- c) Calculate the missile's final velocity

- d) Calculate the range of this missile

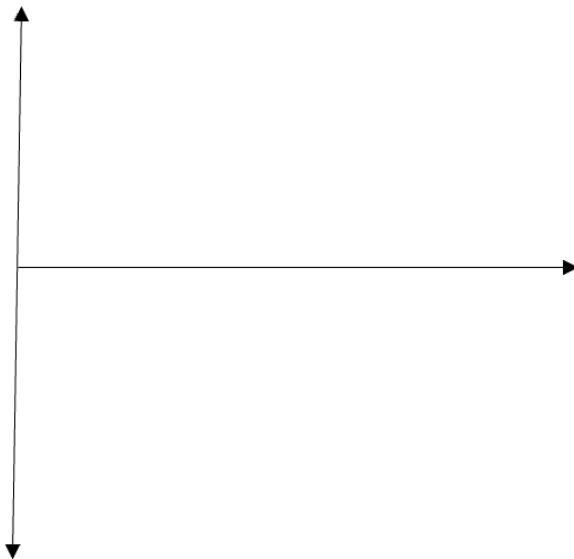
- e) Draw a free body diagram the missile when:
 - i) At takeoff
 - ii) Is at max height
 - iii) It is just before impact

Type 4: Drawing Motion Graphs

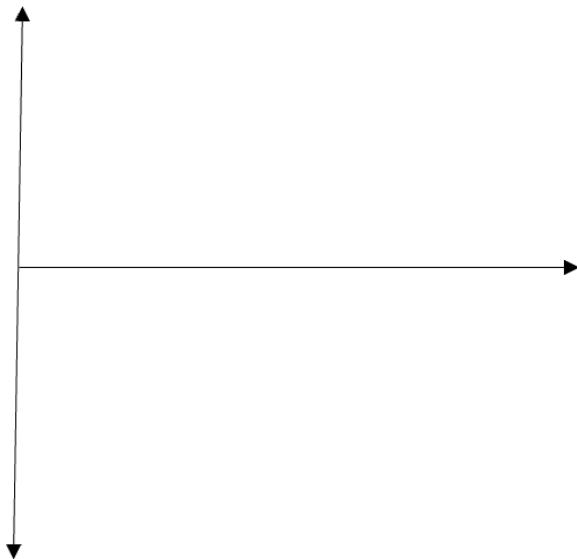
a) Construct a vertical velocity vs time graph for the situation represented in the previous labelling key times and velocities



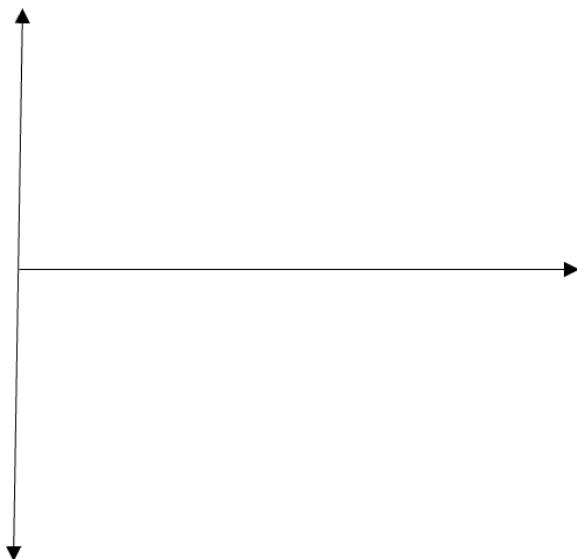
b) Construct a vertical acceleration vs time graph for the situation represented in the previous labelling key times and acceleration



c) Construct a horizontal velocity vs time graph for the situation represented in the previous labelling key times and acceleration



d) Construct a vertical displacement vs time graph for the situation represented in the previous labelling key times and acceleration



Type 5: Maximising Range

Assuming Taylor shot his missile over a flat even surface (not net vertical displacement) determine the optimum angle he should have shot the missile such that the range (horizontal displacement) is maximised. Note: $2\cos\theta\sin\theta = \sin 2\theta$

Proof:

$$\begin{aligned}
 V_y &= U_y - 9.8t \\
 -Usin\theta &= Usin\theta - 9.8t \\
 -2Usin\theta &= -9.8t \\
 t &= \frac{2Usin\theta}{9.8}
 \end{aligned}$$

$$\begin{aligned}
 s_x &= U_x t \\
 s_x/U_x &= t \\
 \frac{s_x}{U\cos(\theta)} &= t
 \end{aligned}$$

$$\begin{aligned}
 s_x/U\cos(\theta) &= 2Usin(\theta)/9.8 \\
 s_x &= 2Usin(\theta) * U\cos(\theta) / 9.8 \\
 &= 2U^2\sin(\theta)\cos(\theta)/9.8 \\
 &= U^2\sin 2\theta/9.8
 \end{aligned}$$

- Max value of sin ratio is 1; this occurs at $\sin 90^\circ \therefore \sin(2\theta) = \sin(90^\circ)$ for max range hence $90 = 2\theta; \theta = 45^\circ$



Type 6: Simultaneous Equations

A golfer hits a ball at 25 m/s at 33.0° to the horizontal on flat ground. It travels 49.3m. She wants to hit a hole 60m away, so she increases the angle at which she hits the ball without changing the launch speed. Determine the smallest increase of angle that allows her to reach the target.

(Hint: $2\cos\theta\sin\theta = \sin 2\theta$)

Air Resistance in Projectile Motion

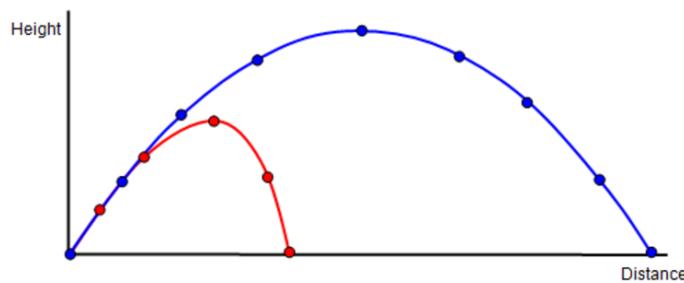
Air Resistance

- This is the ONLY THEORY tested for projectile motion, while we usually ignore the effects of air resistance in our calculations it is highly foolish to ignore the theory for it
- **Air Resistance:** Air resistance/drag is a force acting opposite to the relative motion of any object. Drag force is given by the equation:
$$F_D = \frac{1}{2} \rho v^2 C_D A$$

F_D = drag
 ρ = density of fluid
 v = speed of the object relative to the fluid
 C_D = drag coefficient
 A = cross sectional area
- Therefore, factors affecting drag force experienced by body involved: speed of body, surface area of body and density of surrounding fluid
- When a ball is dropped it experiences a gravitational force downwards and a drag force upwards; eventually these two forces will be equal in magnitude and net force acting on body = 0N. Therefore body will no longer accelerate ($F=ma$; $F=0N$, therefore $a=0m/s^2$) and travel at a constant speed. This constant speed is known as terminal velocity

Effects of Air Resistance

	Relevant Physics Principle
Height Attained	
Range of Projectile	
Angle of Descent	
Upwards vs Downwards Time	



For FULL marks when drawing S_y vs S_x graph:

- 1) Show decrease in max height
- 2) Show decrease in range
- 3) Show a steeper angle of descent
- 4) Show non-parabolic path