VISIONARY TUTORING

Year 12 Methods ATAR:
Differentiation

Product rule, quotient rule, chain rule
Second derivatives

Name: ____________________

Differentiation

Finding the instantaneous rate of change in a function based on one of its variables.

Second Derivatives

Used to determine the *concavity* of a point in a graph.
Negative value: concave up
Positive value: concave down

Function: $y = \mathrm{ax^n}$

Derivative function: $\frac{\mathrm{dy}}{\mathrm{dx}} = anx^{n-1}$

Second derivative function: $\frac{\mathrm{d^2y}}{\mathrm{dx^2}} = an(n-1)x^{n-2}$

OR

The function after being derived twice.
e.g.

$y = 5x^3 \;\; -> \;\; \frac{dy}{dx} = 15x^2 \;\; -> \;\; \frac{d^2y}{dx^2} = 30x$

Apply – Calc Free

Q1. Find the second derivative of the following functions

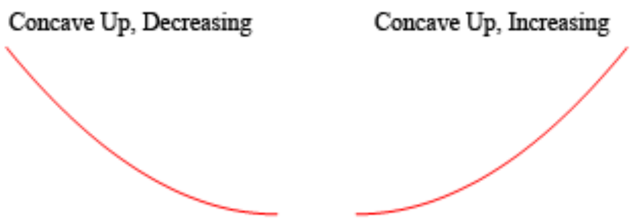
$y = 3x^4 + 10x$

$y = 5x^{\frac{1}{2}}$

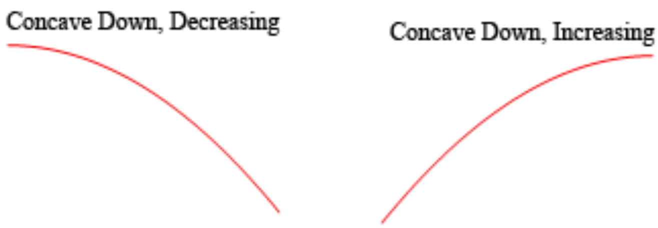
$y = \frac{3}{5x^2}$

Applying the second derivative

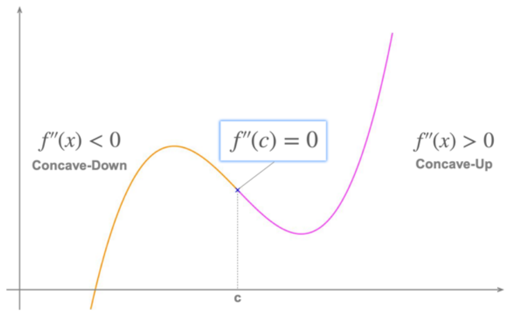
If the second derivative is positive at a point, the shape of the graph at that point is concave up – when the gradient is increasing in value



If the second derivative is negative at a point, the shape of the graph at that point is concave down – when the gradient is decreasing in value



If the second derivative is 0 at a point, that point is a *point of inflection* – the point where *concavity changes*
Further elaborated in Chapter 2



Apply – Calc Free

Determine the concavity of the graph, $y = \frac{1}{x^2}$, when $x = 2$

Find the coordinates for the point of inflection of the graph $y = x^3 + 2x^2 - x$

Product Rule

Used to differentiate a function consisting of the product of two functions

e.g. $y = (x+2)(x+3)$

Function 1: $x+2$
Function 2: $x+3$

The Product Rule Formula:

$y = uv$ [Where u and v represent the 2 functions]

$\frac{dy}{dx} = uv' + u'v$

OR

$y = f(x)g(x)$

$\frac{dy}{dx} = f(x)g'(x) + f'(x)g(x)$

Example 1:

$y = (5x-1)(2x+3)$ [Identify *u* and *v*]

$\frac{dy}{dx} = (5x-1)(2) + (5)(2x+3)$

Simplify…

$\frac{dy}{dx} = 20x + 13$

Example 2 (work through):

$y = (3x-5)(x^2+5x-7)$

Apply – Calc Free

Q2. Differentiate the following functions with respect to x

$y = (x - 10)(x^2 + 8)$

$y = \sqrt{x^3} \times (2x + 1)$

Q3. Find the equation of the tangent to the curve $y = (3x - 5)(4x + 2)$ at the point (2,4)

Q4. Consider the following function with unknown k: $f(x) = (x^2 - 1)(2 - kx^3)$
Determine the value of k if $f'(x) = 5x^4 - 3x^2 + 4x$

Q5. Using the table of values, find the gradient of the function $f(x) = g(x)h(x)$ when $x = 3$

	$x = 1$	$x = 2$	$x = 3$
$g(x)$	5	2	-5
$h(x)$	6	3	4
$g'(x)$	6	-5	-1
$h'(x)$	2	5	7

Quotient Rule

Used to differentiate functions consisting of the ratio of two functions

e.g. $y = \frac{3x-5}{5x-7}$

Function 1: $3x - 5$
Function 2: $5x - 7$

The Quotient Rule Formula:

$$y = \frac{u}{v}$$
$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2}$$

OR

$$y = \frac{f(x)}{g(x)}$$
$$\frac{dy}{dx} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

Example 1:

$$y = \frac{(3x-5)}{(5x-7)}$$

[Identify *u* and *v*]

$$\frac{dy}{dx} = \frac{(3)(5x-7) - (3x-5)(5)}{(5x-7)^2}$$

Simplify…

$$\frac{dy}{dx} = \frac{4}{(5x-7)^2}$$

Example 2 (work through):

$$y = \frac{3x}{x^2+3}$$

Apply – Calc Free

Q6. Differentiate the following with respect to x

$$y = \frac{6x}{5x+1}$$

$$f(x) = \frac{10x^2+2}{8x+1} + 10x^2$$

$$y = \frac{\sqrt{x+1}}{\sqrt{x}}$$

Q7. Determine the coordinates for any points on the curve $y = \frac{2x-1}{5-4x}$ where the gradient is equal to 6

Q8. Using the table of values, find the gradient of the function $f(x) = \frac{g(x)}{h(x)}$ when $x = 1$

	$x = 1$	$x = 2$	$x = 3$
$g(x)$	5	2	-5
$h(x)$	6	3	4
$g'(x)$	6	-5	-1
$h'(x)$	2	5	7

Chain Rule

Used to differentiate a composite function – a function within a function

e.g. $y = 2(2x+3)^2$

Function 1: $2u^2$
Function 2: $x+3$

The Chain Rule Formula:

$y = f(u), u = g(x)$
$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

OR

$y = f(g(x))$
$$\frac{dy}{dx} = f'\big(g(x)\big) \times g'(x)$$

Alternate method:
Treat like power rule, then multiply by derivative of inside function

$y = [f(x)]^n$

$$\frac{dy}{dx} = n[f(x)]^{n-1} \times f'(x)$$

Example 1:

$y = 2(2x+3)^2$
$$\frac{dy}{dx} = 4(2x+3)(2)$$

Simplify…

$$\frac{dy}{dx} = 16x + 24$$

Example 2 (work through):
Find $\frac{dy}{dx}$, in terms of x, given that $y = 7u - 3$, and $u = 2x - 3$

Apply – Calc Free

Q9. Differentiate the following functions with respect to x

$y = (2x - 3)^3$

$y = 5(3x + 4)^5 + 5x^2$

$f(x) = 2(x^4 + x)^8$

Q10. Find $\frac{dy}{dx}$, in terms of x, given that $y = u^2 + 3$, and $u = 3x^2 + x + 1$

Q11. Find the gradient of the function $y = x^2 + (x - 1)^5$ at the point (2, 5)

Q12. Using the table of values, find the gradient of the function $f(x) = g\left(h(x)\right)$ when $x = 2$

	$x = 1$	$x = 2$	$x = 3$
$g(x)$	5	2	-5
$h(x)$	6	3	4
$g'(x)$	6	-5	-1
$h'(x)$	2	5	7